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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
HDL-TR-1830	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER	
A. TITLE (and Subtitle)  Extensions of Models for Transistor Failure Probability Due to Neutron		Technical Reports	
Fluence		6. PERFORMING ORG. REPORT NUMBER  8. CONTRACT OR GRANT NUMBER(s)	
Joseph V. Michalowicz George A. Ausman, Jr	15	MIPR-7655Ø	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Harry Diamond Laboratories 2800 Powder Mill Road Adelphi, MD 20783		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS  Program Ele: 6.27.04.H  Work Unit: 09	
11. CONTROLLING OFFICE NAME AND ADDRESS Director Defense Nuclear Agency Washington, DC 20305	11	Apr 78 78 25 12 21 p	
14. MONITORING AGENCY NAME & ADDRESS(II different	FP34/	UNCLASSIFIED	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
17. DISTRIBUTION STATEMENT (of the abetract entered			
18. SUPPLEMENTARY NOTES HDL Project:  DRCMS Code:  This research was sponsore  NWER subtask V99QAXNFO34, Wor  Agreed Data Base.  19. KEY WORDS (Continue on reverse side if necessary are	697000.22.114 d by the Defer k Unit 09, Mil	nse Nuclear Agency under Litary Equipment Response	
Bipolar transistor Neutron fluence Transistor failure probabilit Circuit susceptibility assess	Equipm Nuclea y Probak ment	ment hardening ar vulnerability bility models	
20. ABSTRACT (Continue on reverse side if necessary and			
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#### 1. INTRODUCTION

Modern tactical warfare increasingly uses electronic equipment of advanced design. Although radiation threat levels at the tactical level are relatively low compared to those at which strategic or space electronic systems must operate, the use of semiconductor components requires that nuclear vulnerability be assessed and the equipment hardened, if necessary. Statistical methods have been developed to provide rapid susceptibility assessments for circuits containing bipolar transistors. These techniques use either neutron-effects or manufacturer's data to predict probability of survival as a function of neutron fluence. However, these methods, which were developed primarily for assessment of new equipment, tend to include large errors of the second kind; that is, they will often predict that a circuit will fail where in reality it would not fail. In order to use these methods for vulnerability assessments whose goal is to determine expected values for circuit failure, it is necessary to extend the models to account for the distributions of the initial device parameters. Additionally, the current models are applicable only to the case where one transistor determines circuit performance. When circuit performance is determined by a critical transistor combination, a Monte Carlo analysis, generally using a network-analysis code, is presently required to determine circuit performance.

Two statistical models have been formulated to predict neutron damage to a single bipolar transistor; one, the Electrical Parameters Model, requires only the electrical parameters given in the manufacturer's specifications, while the other, the Neutron-Effects Data Model, makes use of the radiation-effects data base developed by the Defense Nuclear Agency (DNA) and Harry Diamond Laboratories (HDL). Both are derived from the Messenger-Spratt equation.<sup>2</sup>

$$\frac{1}{\beta_{\phi}} = \frac{1}{\beta_{0}} + C\phi \tag{1}$$

$$=\frac{1}{\beta_0}+\frac{K}{f_T}\phi,\tag{2}$$

where

 $\beta_0$  = initial current gain (before irradiation),

 $\beta_{\phi}$  = final current gain (after irradiation),

 $\phi$  = neutron fluence,

C = neutron damage factor,

 $f_T$  = transistor gain-bandwidth product, and

 $K = f_T C$  = normalized neutron damage constant.

The Electrical Parameters Model is applicable to bipolar transistors in general; the formula for the probability of transistor failure is derived from equation (2), with K treated as a lognormal random variable and  $\phi$ ,  $f_T = f_{T \min}$  (minimum specified value for  $f_T$ ), and  $\beta_0 = \beta_{\min}$  (minimum specified current gain at the peak of the gain versus current curve) as constants.

The Neutron-Effects Data Model is appropriate when neutron-irradiation test data are

<sup>&</sup>lt;sup>1</sup> D. L. Durgin, D. R. Alexander, and R. N. Randall, Hardening Options for Neutron Effects, Harry Diamond Laboratories CR-74-052-1 (15 November 1976).

<sup>&</sup>lt;sup>2</sup> G. C. Messenger and J. P. Spratt, The Effects of Neutron Irradiation on Germanium and Silicon, Proc. IRE, 46 (1958), 1938.

available. It is based on the rearranged expression

$$\beta_{\phi} = \frac{1}{(A+\phi)C} = \frac{A\beta_0}{A+\phi},\tag{3}$$

where  $A = (f_T/\beta_0)/K$  and  $\phi$  are treated as constants and C as a lognormal random variable. Since

$$A = \frac{1}{C\beta_0} \tag{4}$$

or

$$\beta_0 = \frac{1}{AC},$$

the probability of failure can be expressed in terms of the distribution of the random variable  $\beta_0$ , which is also lognormal since  $\ln \beta_0 = -\ln C - \ln A$  is normal. Note that, in contrast to the Electrical Parameters Model, where  $\beta_0$  and K (or C) may be treated as independent, the requirement that A be constant in the Neutron-Effects Data Model leads to equation (4), which relates  $\beta_0$  and C. This equation then determines the relationship between the probability distributions of  $\beta_0$  and C.

The Electrical Parameters Model was found to be extremely conservative in its prediction of failure, mainly because it treats  $\beta_0 = \beta_{\min}$  and  $f_T = f_{T\min}$  as constants. This approach is helpful when designing or hardening equipment; however, in a vulnerability analysis it leads to large errors of the second kind (i.e., the circuit is predicted to be soft when it is in fact hard). Also, both models handle two-transistor combinations through a Monte Carlo approach only.

In the present work these models for transistor-failure probability are extended to include probability distributions for the initial current gains and to allow nonzero origins for all random variables concerned. In fact, the formulas developed can be readily adapted to any type of input distributions. Further, these models are generalized to consider two-transistor combinations. Test cases are calculated to compare the failure probability curves generated by these models with previous results.

# 2. ELECTRICAL PARAMETERS MODEL, WITH $\beta_0$ AS RANDOM VARIABLE

Consider the expression

$$\frac{1}{\beta_{\phi}} = \frac{1}{\beta_0} + C\phi,\tag{5}$$

where  $\phi$  is constant, and  $\beta_0$  and C are lognormal random variables as shown in figures 1 and 2. (Unless otherwise specified, all constants are assumed to be positive.) Now  $\beta_0$  and C are assumed to be independent. If this is not the case, the joint density function of  $\beta_0$  and C would be needed; with this the ensuing analysis would remain valid. Another approach is available when there is a high degree of correlation between  $1/\beta_0$  and C.

Standard techniques for finding the distribution of a function of two random variables3 will

<sup>&</sup>lt;sup>1</sup> D. L. Durgin, D. R. Alexarder, and R. N. Randall, Hardening Options for Neutron Effects, Harry Diamond Laboratories CR-74-052-1 (15 November 1976), p. III-49.

<sup>&</sup>lt;sup>3</sup> Paul G. Hoel, Introduction to Mathematical Statistics, 4th Edition, John Wiley & Sons, Inc. (1971), p. 249 f.

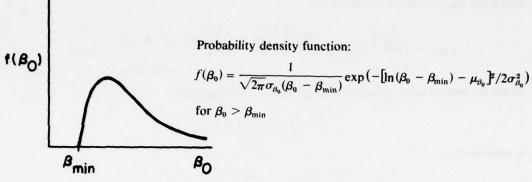


Figure 1. Lognormal distribution for  $\beta_0$ .

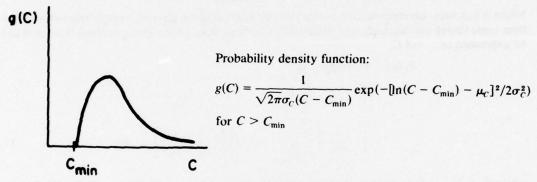


Figure 2. Lognormal distribution for C.

be used in determining the probability density  $p(\beta_{\phi})$ . Rearranging equation (5) gives

$$\beta_{\phi} = \frac{\beta_0}{1 + \phi \beta_0 C}$$

from which it is clear that  $\beta_{\phi}$  is a decreasing function of C when  $\beta_0$  is held fixed.

Then the joint density of  $\beta_0$  and  $\beta_{\phi}$  is given by

$$h(\beta_0, \beta_{\phi}) = \frac{f(\beta_0)g(C)}{\left|\frac{\partial \beta_{\phi}}{\partial C}\right|} \begin{vmatrix} c = \frac{1}{\phi} \left(\frac{1}{\beta_{\phi}} - \frac{1}{\beta_{\phi}}\right) \\ c = \frac{f(\beta_0)g(C)}{\left[\frac{\phi \beta_0^2}{(1 + \phi \beta_0 C)^2}\right]} \end{vmatrix} c = \frac{f(\beta_0)g\left[\frac{1}{\phi} \left(\frac{1}{\beta_{\phi}} - \frac{1}{\beta_0}\right)\right]}{\phi \beta_{\phi}^2}$$

This gives the probability density of  $\beta_{\phi}$ 

$$p(\beta_{\phi}) = \begin{cases} \frac{1}{\phi \beta_{\phi}^{2}} \int_{\frac{\beta_{\phi}}{1 - \phi C_{\min} \beta_{\phi}}}^{\infty} f(\beta_{0}) g \left[ \frac{1}{\phi} \left( \frac{1}{\beta_{\phi}} - \frac{1}{\beta_{0}} \right) \right] d\beta_{0} & \text{if } 0 < \beta_{\phi} \leq \frac{1}{C_{\min} \phi} \\ 0 & \text{otherwise} \end{cases}$$

$$(6)$$

where the lower limit of integration arises because the inequality

$$C = \frac{1}{\phi} \left( \frac{1}{\beta_{\phi}} - \frac{1}{\beta_{0}} \right) \ge C_{\min}$$

is equivalent to

$$\beta_0 \geq \frac{\beta_\phi}{1 - \phi C_{\min} \beta_\phi}.$$

When it has been determined that a given circuit will fail if the gain of a single transistor is less than some threshold value,  $\beta_T$ , the probability of circuit failure for a given neutron fluence  $\phi$  can be expressed as

$$P_{F}(\phi) = \text{Prob} (\beta_{\phi} < \beta_{T})$$

$$= \int_{0}^{\beta_{T}} p(\beta_{\phi}) d\beta_{\phi}$$

$$= \int_{0}^{\beta_{T}} \frac{1}{\phi \beta_{\phi}^{2}} \int_{\frac{\beta_{\phi}}{1 - \phi C_{min} \beta_{\phi}}}^{\infty} f(\beta_{0}) g \left[ \frac{1}{\phi} \left( \frac{1}{\beta_{\phi}} - \frac{1}{\beta_{0}} \right) \right] d\beta_{0} d\beta_{\phi}$$

$$(7)$$

where  $\beta_T < 1/(C_{\min}\phi)$  represents the transistor gain threshold for circuit failure. (If  $\beta_T \ge 1/(C_{\min}\phi)$ , then  $P_F(\phi) = 1$ .)

However, in the HONE Electrical Parameters Model, the normalized variable  $K = f_T C$  is used in place of C with  $f_T$  constant (equal to  $f_{T \min}$ ), but this change is readily incorporated into our derivation. Since C is a lognormal variable, K is also, since  $\ln(K - K_{\min}) = \ln(C - C_{\min}) + \ln f_T$  is normal, where  $K_{\min} = f_T C_{\min}$ . In fact, it is clear that if the probability density function for K is written as

$$g_1(K) = \frac{1}{\sqrt{2\pi}\sigma_K(K - K_{\min})} \exp(-[\ln(K - K_{\min}) - \mu_K]^2/2\sigma_K^2)$$
 for  $K > K_{\min}$ ,

then we must have

$$\mu_K = \mu_C + \ln f_T, \sigma_K = \sigma_C,$$

and so

$$g_1(K) = \frac{1}{f_T} g\left(\frac{K}{f_T}\right),\,$$

where g is the probability density function for C. The probability of failure is then given by

$$P_{F}(\phi) = \int_{0}^{\beta_{T}} \frac{f_{T}}{\phi \beta_{\phi}^{2}} \int_{\frac{\beta_{\phi} f_{T}}{f_{T} - \phi K_{\min} \beta_{\phi}}}^{\infty} f(\beta_{0}) g_{1} \left[ \frac{f_{T}}{\phi} \left( \frac{1}{\beta_{\phi}} - \frac{1}{\beta_{0}} \right) \right] d\beta_{0} d\beta_{\phi}$$
 (8)

with  $\beta_T < f_T/(K_{\min}\phi)$  as the threshold gain. This same result may be obtained by applying the preceding techniques to the expression

$$\beta_{\phi} = \frac{f_T}{\frac{f_T}{\beta_0} + K\phi},$$

with  $\beta_0$  and K independent lognormal random variables and  $\phi$  and  $f_T$  constant.

# 3. NEUTRON-EFFECTS DATA MODEL—CASE OF TWO TRANSISTORS

The following formula is used for a single transistor<sup>1</sup>

$$\beta_{\phi} = \frac{1}{(A + \phi)C},\tag{9}$$

where  $A = (f_T/\beta_0)/K$  and  $\phi$  are constants. Since

$$A = \frac{1}{K} \left( \frac{f_T}{\beta_0} \right) = \frac{1}{C\beta_0} ,$$

we may replace C by  $1/(A\beta_0)$  in equation (9) to obtain

$$\beta_{\phi} = \frac{A\beta_0}{A + \phi} \,. \tag{10}$$

Assume that  $\beta_0$  is a lognormal variable as before. Clearly,  $\beta_{\phi}$  is an increasing function of  $\beta_0$  and so the probability density of  $\beta_{\phi}$  may be obtained from that of  $\beta_0$  as follows.

$$p(\beta_{\phi}) = \frac{f(\beta_{0})}{\left|\frac{d\beta_{\phi}}{d\beta_{0}}\right|} \left|_{\beta_{0} = \frac{A + \phi}{A}\beta_{\phi}}\right|$$

$$= \frac{f(\beta_{0})}{\left(\frac{A}{A + \phi}\right)} \left|_{\beta_{0} = \frac{A + \phi}{A}\beta_{\phi}}\right|$$

$$= \frac{A + \phi}{A} f\left(\frac{A + \phi}{A}\beta_{\phi}\right) \quad \text{for} \quad \beta_{\phi} \ge \frac{A\beta_{\min}}{A + \phi}$$
(11)

<sup>&</sup>lt;sup>1</sup> D. L. Durgin, D. R. Alexander, and R. N. Randall, Hardening Options for Neutron Effects—Final Report, Harry Diamond Laboratories CR-74-052-1 (15 November 1976).

Note that

Prob 
$$(\beta_{\phi} < \beta_{T}) = \int_{\frac{A\beta_{\min}}{A + \phi}}^{\beta_{T}} p(\beta_{\phi}) d\beta_{\phi}$$

$$= \int_{\frac{A\beta_{\min}}{A}}^{\frac{(A + \phi)\beta_{T}}{A}} \frac{A + \phi}{A} f(\beta_{0}) \frac{A}{A + \phi} d\beta_{0}$$

$$= \int_{\frac{\beta_{\min}}{A}}^{\frac{(A + \phi)\beta_{T}}{A}} f(\beta_{0}) d\beta_{0}$$

$$= \text{Prob} \left(\beta_{0} < \frac{A + \phi}{A} \beta_{T}\right)$$

which, by including a possibly nonzero origin for the distribution of  $\beta_0$ , extends the relationship between distribution functions used by Durgin et al. to develop the Neutron-Effects Data Model.

The HONE model treats one transistor at a time. If there are two transistors in a circuit with failure probabilities  $P_{F_1}(\phi)$  and  $P_{F_2}(\phi)$ , independence of the transistors is assumed and the circuit failure probability is calculated from

$$P_F(\phi) = 1 - [1 - P_{F_a}(\phi)][1 - P_{F_a}(\phi)].$$

This report generalizes the methodology of the Neutron-Effects Data Model and the Electrical Parameters Model to treat the case depicted in figure 3 where a combination of two transistors,  $T_1$  with gain  $\beta_1$  and  $T_2$  with gain  $\beta_2$ , is used in a series voltage regulator circuit to amplify an input current,  $I_i$  (from the sensing circuit) to some final current  $I_f$  (the current necessary to maintain a constant voltage drop across  $R_L$ ). If the gains  $\beta_1$  and  $\beta_2$  decrease because of exposure to neutron radiation, then the sensing circuit increases  $I_i$  to compensate for the gain loss so that  $I_f$  continues to produce the desired voltage. However, practical circuit limitations place a maximum on the value of  $I_i$  that can be input to  $T_1$ . This circuit limitation establishes a

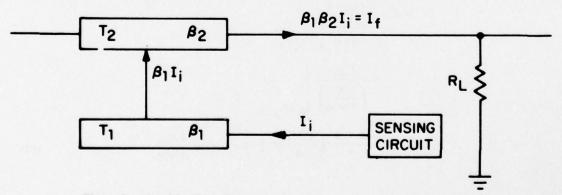


Figure 3. Combination of two transistors in a voltage regulator circuit.

<sup>&</sup>lt;sup>1</sup> D. L. Durgin, P. R. Alexander, and R. N. Randall, Hardening Options for Neutron Effects—Final Report, Harry Diamond Laboratories CR-74-052-1 (15 November 1976).

lower limit,  $\beta_T$ , for the gain product  $\beta_1\beta_2$ , below which the circuit will malfunction. Since we are interested in the operation of the circuit after irradiation, we denote  $\beta_1$  and  $\beta_2$  by  $\beta_{\phi_1}$  and  $\beta_{\phi_2}$ , respectively, and compute the probability of circuit failure  $\operatorname{Prob}(\beta_{\phi_1}\beta_{\phi_2} < \beta_T)$ .

For the Neutron-Effects Data Model, the final current gain is then given by

$$\beta_{\phi} = \beta_{\phi_1} \beta_{\phi_2}$$

$$= \frac{A_1 \beta_{01}}{A_1 + \phi} \frac{A_2 \beta_{02}}{A_2 + \phi} = \frac{A_1 A_2 \beta_{01} \beta_{02}}{(A_1 + \phi)(A_2 + \phi)}$$
(12)

where  $\beta_{01}$  and  $\beta_{02}$  are lognormal random variables with probability densities

$$f_1(\beta_{01}) = \frac{1}{\sqrt{2\pi}\sigma_{\beta_{01}}(\beta_{01} - \beta_{1\,\text{min}})} \exp(-\left[\ln\left(\beta_{01} - \beta_{1\,\text{min}}\right) - \mu_{\beta_{01}}\right]^2/2\sigma_{\beta_{01}}^2)$$
(13)

for 
$$\beta_{01} \ge \beta_{1 \min}$$
,

$$f_2(\beta_{02}) = \frac{1}{\sqrt{2\pi}\sigma_{\beta_{02}}(\beta_{02} - \beta_{2\min})} \exp(-[\ln(\beta_{02} - \beta_{2\min}) - \mu_{\beta_{02}}]^2 / 2\sigma_{\beta_{02}}^2)$$
(14)

for 
$$\beta_{02} \ge \beta_{2 \min}$$
.

Assume that  $\beta_{01}$  and  $\beta_{02}$  are independent. The expression

$$\beta_{\phi} = \frac{A_1 A_2 \beta_{01} \beta_{02}}{(A_1 + \phi)(A_2 + \phi)}$$

evinces  $\beta_{\phi}$  as an increasing function of  $\beta_{02}$  when  $\beta_{01}$  is held fixed. Computation of the joint density of  $\beta_{01}$  and  $\beta_{\phi}$  yields

$$\begin{split} h(\beta_{01},\beta_{\phi}) &= \frac{f_{1}(\beta_{01})f_{2}(\beta_{02})}{\left|\frac{\partial\beta_{\phi}}{\partial\beta_{02}}\right|} \left|_{\beta_{02} = \frac{(A_{1} + \phi)(A_{2} + \phi)\beta_{\phi}}{A_{1}A_{2}\beta_{01}}} \right| \\ &= \frac{f_{1}(\beta_{01})f_{2}(\beta_{02})}{\left(\frac{A_{1}A_{2}\beta_{01}}{(A_{1} + \phi)(A_{2} + \phi)}\right)} \left|_{\beta_{02} = \frac{(A_{1} + \phi)(A_{2} + \phi)\beta_{\phi}}{A_{1}A_{2}\beta_{01}}} \right| \\ &= \frac{(A_{1} + \phi)(A_{2} + \phi)}{A_{1}A_{2}\beta_{01}} f_{1}(\beta_{01})f_{2}\left(\frac{(A_{1} + \phi)(A_{2} + \phi)\beta_{\phi}}{A_{1}A_{2}\beta_{01}}\right). \end{split}$$

Integrating over  $\beta_{01}$  gives the probability density of  $\beta_{\phi}$ :

$$p(\beta_{\phi}) = \begin{cases} \frac{(A_{1} + \phi)(A_{2} + \phi)}{A_{1}A_{2}} \int_{\beta_{1} \min}^{\frac{(A_{1} + \phi)(A_{2} + \phi)\beta_{\phi}}{A_{1}A_{2}\beta_{2} \min}} \frac{1}{\beta_{01}} f_{1}(\beta_{01}) f_{2} \left[ \frac{(A_{1} + \phi)(A_{2} + \phi)\beta_{\phi}}{A_{1}A_{2}\beta_{01}} \right] d\beta_{01} \\ \text{if } \beta_{\phi} \geq \frac{A_{1}A_{2}\beta_{1} \min \beta_{2} \min}{(A_{1} + \phi)(A_{2} + \phi)}, \\ 0 \quad \text{otherwise.} \end{cases}$$

$$(15)$$

The probability of circuit failure for a specified neutron fluence  $\phi$  and gain threshold  $\beta_T$  is then

The probability of circuit failure for a specified neutron fluence 
$$\phi$$
 and gain threshold  $\beta_T$  is then given by

$$\frac{\left(A_1 + \phi)(A_2 + \phi)}{A_1 A_2} \int_{\frac{A_1 A_2 \beta_{1 \min} \beta_{2 \min}}{(A_1 + \phi)(A_2 + \phi)}}^{\beta_T} \int_{\beta_{1 \min}}^{\frac{(A_1 + \phi)(A_2 + \phi)\beta_{\phi}}{A_1 A_2 \beta_{2 \min}}} \frac{1}{\beta_{01}} f_1(\beta_{01}) f_2 \left[ \frac{(A_1 + \phi)(A_2 + \phi)\beta_{\phi}}{A_1 A_2 \beta_{01}} \right] d\beta_{01} d\beta_{\phi}$$
(16)

if  $\beta_T \ge \frac{A_1 A_2 \beta_{1 \min} \beta_{2 \min}}{(A_1 + \phi)(A_2 + \phi)}$ ,

$$0 \quad \text{if } \beta_T < \frac{A_1 A_2 \beta_{1 \min} \beta_{2 \min}}{(A_1 + \phi)(A_2 + \phi)}.$$

# 4. ELECTRICAL PARAMETERS MODEL—CASE OF TWO TRANSISTORS

#### **Constant Initial Gains**

Suppose two transistors are combined as in the previous section with post-irradiation current gains given by

$$\frac{1}{\beta_{\phi_1}} = \frac{1}{\beta_{01}} + C_1 \phi,$$

$$\frac{1}{\beta_{\phi_2}} = \frac{1}{\beta_{02}} + C_2 \phi,$$

where  $C_1$  and  $C_2$  are independent lognormal random variables with probability densities

$$g_1(C_1) = \frac{1}{\sqrt{2\pi}\sigma_{c_1}(C_1 - C_{\text{tmin}})} \exp\left(-\left[\ln(C_1 - C_{\text{1min}}) - \mu_{c_1}\right]^2 / 2\sigma_{c_1}^2\right) \quad \text{for} \quad C_1 > C_{\text{1min}}, \quad (17)$$

$$g_2(C_2) = \frac{1}{\sqrt{2\pi}\sigma_{c_2}(C_2 - C_{2\min})} \exp\left(-\left[\ln(C_2 - C_{2\min}) - \mu_{c_2}\right]^2 / 2\sigma_{c_2}^2\right) \quad \text{for} \quad C_2 > C_{2\min}, \quad (18)$$

and  $\beta_{01} = \beta_{1 \min}$  and  $\beta_{02} = \beta_{2 \min}$  are constants. Then

$$\beta_{\phi} = \beta_{\phi_1} \beta_{\phi_2}$$

$$= \frac{\beta_{01} \beta_{02}}{1 + \beta_{01} \phi C_1 + \beta_{02} \phi C_2 + \beta_{01} \beta_{02} \phi^2 C_1 C_2}$$
(19)

is clearly a decreasing function of  $C_2$  when  $C_1$  is held fixed. The joint density of  $C_1$  and  $\beta_{\phi}$  is

computed to be

$$\begin{split} h(C_{1},\beta_{\phi}) &= \frac{g_{1}(C_{1})g_{2}(C_{2})}{\left|\frac{\partial\beta_{\phi}}{\partial C_{2}}\right|} \left| C_{z} = \frac{1}{\beta_{0z}\phi} \left[ \frac{\beta_{01}\beta_{0z}}{\beta_{o}(1+\beta_{01}\phi C_{1})} - 1 \right] \\ &= \frac{g_{1}(C_{1})g_{2}(C_{2})}{\left(\frac{\beta_{01}}{1+\beta_{01}\phi C_{1}}\right) \left[\frac{\beta_{02}^{2}\phi}{(1+\beta_{02}\phi C_{2})^{2}}\right]} \left| C_{z} = \frac{1}{\beta_{0z}\phi} \left[ \frac{\beta_{01}\beta_{0z}}{\beta_{o}(1+\beta_{01}\phi C_{1})} - 1 \right] \\ &= \frac{\beta_{01}}{\phi\beta_{\phi}^{2}(1+\beta_{01}\phi C_{1})} g_{1}(C_{1})g_{2} \left( \frac{1}{\beta_{02}\phi} \left[ \frac{\beta_{01}\beta_{02}}{\beta_{\phi}(1+\beta_{01}\phi C_{1})} - 1 \right] \right) \end{split}$$

The probability density of  $\beta_{\phi}$  can then be expressed as

$$p(\beta_{\phi}) = \begin{cases} \frac{\beta_{01}}{\phi \beta_{\phi}^{2}} \int_{C_{1 \min}}^{\frac{1}{\beta_{01} \phi}} \left[ \frac{\beta_{01} \beta_{02}}{\beta_{\phi} (1 + \beta_{02} \phi C_{2 \min})} - 1 \right] \frac{1}{1 + \beta_{01} \phi C_{1}} g_{1}(C_{1}) g_{2} \left[ \frac{1}{\beta_{02} \phi} \left( \frac{\beta_{01} \beta_{02}}{\beta_{\phi} (1 + \beta_{01} \phi C_{1})} - 1 \right) \right] dC_{1} \\ \text{if } 0 < \beta_{\phi} \leq \frac{\beta_{01} \beta_{02}}{(1 + \beta_{01} \phi C_{1 \min})(1 + \beta_{02} \phi C_{2 \min})}, \end{cases}$$
(20)
$$0 \quad \text{otherwise.}$$

The probability of circuit failure in this case then becomes

$$P_{F}(\phi) = \begin{cases} \frac{\beta_{01}}{\phi} \int_{0}^{\beta_{T}} \frac{1}{\beta_{\phi}^{2}} \int_{C_{1 \min}}^{\frac{1}{\beta_{01}\phi}} \left[ \frac{\beta_{01}\beta_{02}}{\beta_{\phi}(1 + \beta_{02}\phi C_{2 \min})}^{-1} \right] \frac{1}{1 + \beta_{01}\phi C_{1}} g_{1}(C_{1}) g_{2} \left( \frac{1}{\beta_{02}\phi} \left[ \frac{\beta_{01}\beta_{02}}{\beta_{\phi}(1 + \beta_{01}\phi C_{1})} - 1 \right] \right) dC_{1} d\beta_{\phi} \end{cases}$$

$$if \quad \beta_{T} \leq \frac{\beta_{01}\beta_{02}}{(1 + \beta_{01}\phi C_{1 \min})(1 + \beta_{02}\phi C_{2 \min})}, \qquad (21)$$

$$1 \quad if \quad \beta_{T} > \frac{\beta_{01}\beta_{02}}{(1 + \beta_{01}\phi C_{1 \min})(1 + \beta_{02}\phi C_{2 \min})},$$

for a specified neutron fluence  $\phi$  and threshold gain  $\beta_T$ .

## 4.2 Variable Initial Gains

Now consider the case where  $\beta_{\phi}$  is as expressed in equation (19) but, in addition to  $C_1$  and  $C_2$ ,  $\beta_{01}$  and  $\beta_{02}$  are lognormal random variables as well, with probability densities as given in equations (13) and (14); assume that all four variables are independent. The technique presented by Papoulis<sup>4</sup> will be used to determine the probability density of  $\beta_{\phi}$ .

<sup>&</sup>lt;sup>4</sup> A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill (1965), pp. 234-235.

The transformation

$$\beta_{\phi} = \frac{\beta_{01}\beta_{02}}{1 + \phi\beta_{01}C_1 + \phi\beta_{02}C_2 + \phi^2\beta_{01}\beta_{02}C_1C_2}$$

$$C_1 = C_1$$

$$C_2 = C_2$$

$$\beta_{01} = \beta_{01}$$

has associated Jacobian

$$J = \begin{vmatrix} \frac{\partial \beta_{\phi}}{\partial C_{1}} \frac{\partial \beta_{\phi}}{\partial C_{2}} \frac{\partial \beta_{\phi}}{\partial \beta_{01}} \frac{\partial \beta_{\phi}}{\partial \beta_{02}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$
$$= -\frac{\partial \beta_{\phi}}{\partial \beta_{02}}$$
$$= \frac{-\beta_{01}}{(1 + \phi \beta_{01} C_{1})(1 + \phi \beta_{02} C_{2})^{2}}.$$

Therefore, the joint density function of  $\beta_{\phi}$ ,  $C_1$ ,  $C_2$ , and  $\beta_{01}$  is given by

Therefore, the joint density function of 
$$\beta_{\phi}$$
,  $C_1$ ,  $C_2$ , and  $\beta_{01}$  is given by
$$h(\beta_{\phi}, C_1, C_2, \beta_{01}) = \frac{g_1(C_1)g_2(C_2)f_1(\beta_{01})f_2(\beta_{02})}{|J|} \left| \beta_{02} = \frac{\beta_{\phi}\left(\frac{1}{\beta_{01}} + C_1\phi\right)}{1 - C_2\phi\beta_{\phi}\left(\frac{1}{\beta_{01}} + C_1\phi\right)} \right|$$

$$= \frac{(1 + \phi\beta_{01}C_1)}{\beta_{01}\left[1 - \phi\beta_{\phi}C_2\left(\frac{1}{\beta_{01}} + C_1\phi\right)\right]^2} g_1(C_1)g_2(C_2)f_1(\beta_{01})f_2 \left[\frac{\beta_{\phi}\left(\frac{1}{\beta_{01}} + C_1\phi\right)}{1 - C_2\phi\beta_{\phi}\left(\frac{1}{\beta_{01}} + C_1\phi\right)}\right]$$

Then  $p(\beta_{\phi})$  is computed as a marginal distribution:

$$\rho(\beta_{\phi}) = \begin{cases} \frac{1}{\int_{(\phi^{1}\beta_{\phi}C_{2\min})}^{1}} g_{1}(C_{1}) \int_{\frac{1}{\phi}}^{\frac{1}{\beta_{\phi}C_{1}\phi^{2}}} g_{1}(C_{1}) \int_{\frac{1}{\phi}}^{\frac{1}{\beta_{\phi}C_{1}\phi^{2}}} \frac{1}{|\beta_{\phi}C_{1}\phi^{2}|} g_{2}(C_{2}) \int_{\frac{\beta_{\phi}C_{2}\phi}{1-\beta_{\phi}C_{1}C_{2}\phi^{2}}}^{\frac{\beta_{\phi}C_{2}\phi}{1-\beta_{\phi}C_{1}C_{2}\phi^{2}}} g_{2}(C_{2}) g$$

The probability of circuit failure is then given by

$$P_{F}(\phi) = \int_{0}^{\beta_{T}} \int_{C_{1 \min}}^{\frac{1}{\phi^{2} \beta_{o} C_{2 \min}}} g_{1}(C_{1}) \int_{\frac{1}{\phi}}^{\frac{1}{\beta_{o} C_{1} \phi^{2}}} \frac{1}{\frac{1}{\phi} \left[ \frac{1}{\beta_{o} (\frac{1}{\beta_{1 \min}} + C_{1} \phi)} - \frac{1}{\beta_{2 \min}} \right]} g_{2}(C_{2})$$

$$\frac{\frac{\beta_{o} \left( \frac{1}{\beta_{2 \min}} + C_{2} \phi \right)}{1 - C_{1} \phi \beta_{o} \left( \frac{1}{\beta_{2 \min}} + C_{2} \phi \right)}}{\frac{1 - C_{1} \phi \beta_{o} \left( \frac{1}{\beta_{2 \min}} + C_{2} \phi \right)}{\beta_{01} \left[ 1 - \phi \beta_{\phi} C_{2} \left( \frac{1}{\beta_{01}} + C_{1} \phi \right) \right]^{2}} f_{1}(\beta_{01}) f_{2} \left[ \frac{\beta_{\phi} \left( \frac{1}{\beta_{01}} + C_{1} \phi \right)}{1 - C_{2} \phi \beta_{\phi} \left( \frac{1}{\beta_{01}} + C_{1} \phi \right)} d\beta_{01} dC_{2} dC_{1} d\beta_{\phi} dC_{2} dC_{1} dC_$$

for a threshold gain  $\beta_T \le 1/(C_{1\min}C_{2\min}\phi^2)$  (otherwise  $P_F(\phi) = 1$ ).

# 5. COMPUTATION OF MODEL PARAMETERS

In general, suppose a sample mean  $\bar{x}$  and sample standard deviation  $s_x$  have been computed for a random variable x known to have a standard lognormal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp[-(\ln x - \mu)^2/2\sigma^2] \quad \text{for} \quad x > 0.$$

For this distribution,

$$m \equiv \text{mean} = \exp\left(\mu + \frac{\sigma^2}{2}\right),$$
  
 $s \equiv \text{standard deviation} = \sqrt{e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)}$   
 $= m\sqrt{e^{\sigma^2} - 1}$ 

To estimate  $\mu$  and  $\sigma$ , set

$$\bar{x} = m = \exp\left(\mu + \frac{\sigma^2}{2}\right),$$

$$s_x = s = \bar{x}\sqrt{e^{\sigma^2} - 1}.$$

Then we get the following formulas from which to compute  $\sigma$  and  $\mu$ :

$$\sigma = \sqrt{\ln\left(1 + \frac{s_x^2}{\tilde{x}^2}\right)} ,$$

$$\mu = \ln \tilde{x} - \frac{\sigma^2}{2}$$

$$= \ln \hat{x} - \ln \sqrt{1 + \frac{s_x^2}{\tilde{x}^2}}$$

$$= \ln\left(\frac{\tilde{x}^2}{\sqrt{\tilde{x}^2 + s_x^2}}\right).$$
(24)

If the origin of the lognormal distribution is shifted to some value, say  $x_{\min}$ , then  $x - x_{\min}$  has a standard lognormal distribution, so these same formulas can be used as long as  $\bar{x}$  is replaced by  $\bar{x} - x_{\min}$ ; that is,

$$\sigma = \sqrt{\ln\left[1 + \frac{s_x^2}{(\bar{x} - x_{\min})^2}\right]} , \qquad (26)$$

$$\mu = \ln \left[ \frac{(\bar{x} - x_{\min})^2}{\sqrt{(\bar{x} - x_{\min})^2 + s_x^2}} \right]. \tag{27}$$

Equations (26) and (27), with  $x = \beta_0$  and x = C (or K), are used to compute the parameters of the lognormal distributions of  $\beta_0$  and C (or K) in the extension of the Electrical Parameters Model in section 2, and to compute  $\mu_{\beta_{01}}$ ,  $\sigma_{\beta_{01}}$ ,  $\mu_{\beta_{02}}$ ,  $\sigma_{\beta_{02}}$ ,  $\mu_{C_1}$ ,  $\sigma_{C_1}$ ,  $\mu_{C_2}$ , and  $\sigma_{C_2}$ , in the two-transistor cases.

The computation of A in the Neutron-Effects Data Model requires the statistics of C as well as  $\beta_0$ . Since  $C = 1/(A\beta_0)$ , we have  $\ln C = -\ln \beta_0 - \ln A$  and  $E[\ln C] = -E[\ln \beta_0] - \ln A$  as the relation between the corresponding expected values. This gives  $A = e^{\ln A} = \exp[-(E[\ln \beta_0] + E[\ln C])]$ . Thus, A can be estimated from

$$A = \exp[-(\overline{\ln \beta_0} + \overline{\ln C})] \tag{28}$$

This formula requires estimation from the data of the means of  $\ln \beta_0$  and  $\ln C$  rather than of  $\beta_0$  and C. However, if  $\beta_0$  and C have *unshifted* lognormal distributions

$$f(\beta_0) = \frac{1}{\sqrt{2\pi}\sigma_{\beta_0}\beta_0} \exp[-(\ln\beta_0 - \mu_{\beta_0})^2/2\sigma_{\beta_0}^2] \quad \text{for} \quad \beta_0 > 0$$

$$g(C) = \frac{1}{\sqrt{2\pi}\sigma_C C} \exp[-(\ln C - \mu_C)^2/2\sigma_C^2] \quad \text{for} \quad C > 0$$

A can be estimated from the statistics of  $\beta_0$  and C themselves. For it is clear that

$$E[\ln \beta_0] = \mu_{\beta_0}$$

$$E[\ln C] = \mu_C$$

$$\mu_C = -\mu_{\beta_0} - \ln A$$

 $\sigma_C = \sigma_{\beta_0}$ .

Therefore, we have

so that

$$A = \exp[-(E[\ln \beta_0] + E[\ln C])]$$

$$= \exp[-\mu_{\beta_0} - \mu_C]$$

$$= \frac{\exp(\sigma_{\beta_0}^2)}{\exp(\mu_{\beta_0} + \frac{1}{2}\sigma_{\beta_0}^2)\exp(\mu_C + \frac{1}{2}\sigma_C^2)}$$

$$= \frac{1}{m_{\beta_0}m_C} \left[ \left( \frac{s}{m_{\beta_0}} \right)^2 + 1 \right],$$

where s is the standard deviation of the lognormal distribution of  $\beta_0$ . Consequently, in this case an estimate of A may be made from

$$A = \frac{1}{\overline{\beta_0}\overline{C}} \left[ \left( \frac{s_{\beta_0}}{\overline{\beta_0}} \right)^2 + 1 \right], \tag{29}$$

where  $\bar{\beta}_0$  and  $s_{B_0}$  are the sample mean and sample standard deviation of  $\beta_0$ , and  $\bar{C}$  is the sample mean of C. This formula agrees, except in the notation used, with that given by Durgin et al.<sup>1</sup>

#### 6. RESULTS OF CASE STUDIES

Test cases were examined for each of the model extensions derived. The numerical values of the statistical parameters (tables I and II) were obtained from manufacturer's specifications or from the DNA/HDL radiation-effects data base. The following cases were considered.

- 1. Extended Electrical Parameters Model applied to a single transistor (2N1485)— $\beta_0$  and C lognormal random variables.
- 2. Shifted HONE Electrical Parameters Model derived for a two-transistor combination (2N1183 and 2N697)— $C_1$  and  $C_2$  lognormal random variables,  $\beta_{01} = \beta_{1 \min}$  and  $\beta_{02} = \beta_{2 \min}$ .
- 3. Same as case 2, but with a different value of  $\beta_{1 \text{ min}}$ . This shows the sensitivity of the HONE Model to the selection of  $\beta_{\text{min}}$ .
- 4. Neutron-Effects Data Model derived for a two-transistor combination (2N1183 and 2N697)— $\beta_{01}$  and  $\beta_{02}$  lognormal variables.  $A_1$  and  $A_2$  are computed from  $C_1$ ,  $C_2$ ,  $\beta_{01}$ , and  $\beta_{02}$  data.
- 5. Extended Electrical Parameters Model applied to a two-transistor combination (2N1183 and 2N697)— $\beta_{01}$ ,  $\beta_{02}$ ,  $C_1$ , and  $C_2$  all lognormal random variables.

<sup>&</sup>lt;sup>1</sup> D. L. Durgin, D. R. Alexander, and R. N. Randall, Hardening Options for Neutron Effects—Final Report, Harry Diamond Laboratories CR-74-052-1 (15 November 1976).

TABLE I Input Parameters for Single-Transistor Cases

Case 1		
Extended Electrical Parameters Model with $\beta_0$ and $C$ variable	HONE Electrical Parameters Model	Shifted HONE Electrical Parameters Model
$ \hat{\beta}_{0} = 49.1  s_{\beta_{0}} = 6.10  \beta_{\min} = 35  \tilde{C} = 2.42 \times 10^{-14}  s_{C} = 3.77 \times 10^{-15}  C_{\min} = \tilde{C} - 3s_{C} = 1.289 \times 10^{-14}  \beta_{T} = 10 $	$eta_0 = eta_{\min} = 35$ $C = 2.42 \times 10^{-14}$ $s_C = 3.77 \times 10^{-15}$ $C_{\min} = 0$ $eta_T = 10$	$ \beta_0 = \beta_{\min} = 35 $ $ \tilde{C} = 2.42 \times 10^{-14} $ $ s_C = 3.77 \times 10^{-15} $ $ C_{\min} = 1.289 \times 10^{-14} $ $ \beta_T = 10 $

Note: Single transistor—2N1485;  $\phi$  is in n/cm<sup>2</sup>, C in cm<sup>2</sup>/n, and  $\beta_0$  is dimensionless.

TABLE II
Input Parameters for Two-Transistor Cases

	input Parameters for	I wo- Fransistor Cases					
Case 2  Shifted HONE Electrical Parameters Model Two-Transistor Combination $C_1$ , $C_2$ variable; $\beta_{01}$ , $\beta_{02}$ constant		Case 3  Shifted HONE Electrical Parameters Model Two-Transistor Combination $C_1$ , $C_2$ variable; $\beta_{01}$ , $\beta_{02}$ constant					
				$T_1$ : 2N1183 $\bar{C}_1 = 3.0 \times 10^{-15}$ $s_{C_1} = 4.0 \times 10^{-16}$ $C_{1 \min} = \bar{C}_1 - 3s_{C_1}$ $= 1.8 \times 10^{-15}$ $\beta_{01} = \beta_{1 \min} = 20$	$T_2$ : 2N697 $\bar{C}_2 = 1.8 \times 10^{-15}$ $s_{C_2} = 2.3 \times 10^{-16}$ $C_{2 \min} = \bar{C}_2 - 3s_{C_2}$ $= 1.11 \times 10^{-15}$ $\beta_{02} = \beta_{2 \min} = 40$	Same input parameters $\beta_{1 \text{ min}} = 3$	meters as Case 2 0
					= 666		
Case 4' Neutron-Effects Data Model Two-Transistor Combination		Ca	se 5				
		Extended Electrical Parameters Model Two-Transistor Combination $\beta_{01}$ , $\beta_{02}$ , $C_1$ , $C_2$ variable					
T <sub>1</sub> : 2N1183	T <sub>2</sub> : 2N697	$T_1$ : 2N 1183	$T_2$ : 2N697				
$ \overline{\ln C_1} = -33.495  \overline{\beta}_{01} = 55.4  s_{\beta_{01}} = 17.9  \overline{\beta}_{1 \min} = 20  \overline{\ln \beta_{01}} = 3.967  A_1 = 6.666 \times 10^{12}  \overline{\beta}_{T} = 6.666 \times 10^{12} $	$ \overline{\ln C_2} = -33.959  \overline{\beta}_{02} = 77.0  s_{\beta_{02}} = 10.8  \beta_{2 \min} = 40  \overline{\ln \beta_{02}} = 4.334  A_2 = 7.3447 \times 10^{12}  = 666 $	$ \begin{aligned} \bar{\beta}_{01} &= 55.4 \\ s_{\beta_{01}} &= 17.9 \\ \beta_{1 \min} &= 20 \\ \bar{C}_{1} &= 3.0 \times 10^{-15} \\ s_{C_{1}} &= 4.0 \times 10^{-16} \\ C_{1 \min} &= 1.8 \times 10^{-15} \\ \beta_{T} &= 6 \end{aligned} $	$ \bar{\beta}_{02} = 77.0  s_{\beta_{02}} = 10.8  \beta_{2 \min} = 40  \bar{C}_{2} = 1.8 \times 10^{-15}  s_{C_{2}} = 2.3 \times 10^{-16}  C_{2 \min} = 1.11 \times 10^{-15} $				

Note:  $\phi$  is in n/cm<sup>2</sup>,  $C_1$  and  $C_2$  in cm<sup>2</sup>/n,  $\beta_{01}$  and  $\beta_{02}$  dimensionless, and  $A_1$  and  $A_2$  in n/cm<sup>2</sup>.

Figure 4 indicates the significance of the model extension for the one-transistor case that has been developed in this paper. There are two aspects of the improved one-transistor model developed here which contribute to this difference: (1)  $\beta_0$  is a random variable rather than a constant, and (2) the lognormal distribution for C is allowed to have an origin other than at zero. Figure 4 indicates that the major contributor to this difference is the incorporation of the probability distribution of  $\beta_0$ , because the middle curve, which represents the results obtained by the shifted HONE Model ( $\beta_0$  constant), is practically indistinguishable from the curve for the unshifted HONE Model.

The probability-of-survival curves for the two-transistor cases are shown in figure 5. The curves for cases 2, 3, and 4 were obtained by numerically integrating the appropriate expression for  $P_F(\phi)$  for a sequence of values of  $\phi$ . However, in case 5,  $P_F(\phi)$  is given by a four-fold integral—equation (23)—which does not lend itself to computer evaluation. This curve was developed by a Monte Carlo technique: for each value of  $\phi$ , the distributions of  $\beta_{01}$ ,  $\beta_{02}$ ,  $C_1$ , and  $C_2$  were sampled  $N_{\phi}$  times; for each sample,  $\beta_{\phi}$  was calculated from equation (19) and compared to  $\beta_T$ ; the number of times that  $\beta_{\phi}$  was less than  $\beta_T$  was counted. This count divided by  $N_{\phi}$  then

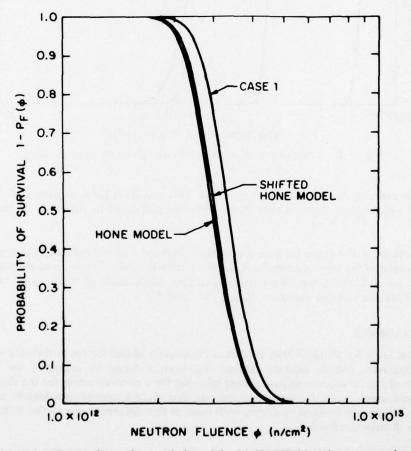


Figure 4. Comparison of extended model with HONE Model—one transistor.

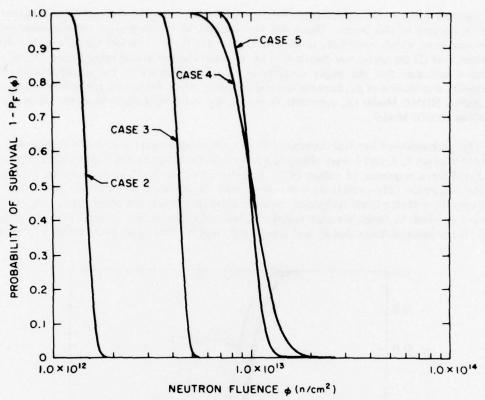


Figure 5. Comparison of models for two-transistor combination.

provided an estimate for  $P_F(\phi) = \text{Prob}(\beta_{\phi} < \beta_T)$ . This was done for a sequence of values of  $\phi$ , with  $N_{\phi}$  chosen large enough (usually  $N_{\phi} = 2000$  was sufficient) to ensure convergence of the estimate.

The position of the curve for case 2 compared to that for case 3 indicates the importance of proper selection of the input parameter  $\beta_{1 \min}$  in the HONE Model. It was to be expected that the curves for cases 4 and 5 would compare favorably, since each of these cases includes the statistics of all four random variables  $\beta_{01}$ ,  $\beta_{02}$ ,  $C_1$ , and  $C_2$ .

# 7. CONCLUSIONS

The conservatism of the HONE Electrical Parameters Model for the probability of failure of a single transistor due to neutron fluence has been reduced by including the probability distribution of the initial current gain  $\beta_0$  and allowing for a nonzero origin for the distribution of the neutron-damage factor C. Test cases indicate that these improvements lead to a significant change in the survival probability curve, with most of this difference due to the inclusion of the probability distribution for  $\beta_0$ .

The HONE model for probability of failure due to neutron fluence has been extended to

consider a circuit whose function depends on the product of two transistor gains. The following models have been derived for this two-transistor combination.

- 1. Electrical Parameters Model, with neutron-damage factors variable and constant initial current gains
- 2. Extended Electrical Parameters Model, with neutron-damage factors and initial current gains all variable
  - 3. Neutron-Effects Data Model

In all these models nonzero origins for the distributions of the random variables concerned are allowed.

Finally, formulas have been derived to estimate the parameters of the probability distributions from transistor test data.

# **ACKNOWLEDGMENT**

The authors would like to acknowledge the many helpful suggestions provided by William E. Sweeney, Jr., William L. Vault, and Paul A. Trimmer during the course of this study.

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